Spin-orbit and orbit-orbit strengths for radioactive neutron-rich doubly magic nucleus ^{132}Sn in relativistic mean field theory

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Relativistic mean field (RMF) theory is applied to investigate the properties of the radioactive neutron-rich doubly magic nucleus ^{132}Sn and the corresponding isotopes and isotones. The two-neutron and two-proton separation energies are well reproduced by the RMF theory. In particular, the RMF results agree with the experimental single-particle spectrum in ^{132}Sn as well as the Nilsson spin-orbit parameter C and orbit-orbit parameter D thus extracted, but remarkably differ from the traditional Nilsson parameters. Furthermore, the present results provide a guideline for the isospin dependence of the Nilsson parameters.

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The concept of magic numbers is one of the most fundamental ingredients for understanding the nature of atomic nuclei. Due to the strong spin-orbit couplings, the magic numbers for stable nuclei are shown as 2, 8, 20, 28, 50, and 82 for both protons and neutrons as well as 126 for neutrons [1, 2], which are no longer simply the shell-closure of the harmonic oscillators. Thus, it is quite sophisticated to predict the next proton and neutron magic numbers [3, 4], which are, nevertheless, critical for guiding the superheavy element synthesis. The phenomenon of shell-closure is also crucial for determining the waiting-points of the rapid neutron-capture process (r-process), which is responsible for the production of more than half of the elements heavier than iron.

With both proton and neutron magic numbers, the so-called doubly magic nuclei form a very small and exclusive club. These nuclei are rigidly spherical and particularly stable compared to their neighbors. Along the β stability line, only five nuclei are included, i.e., ^{4}He, ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb. Furthermore, by simply combining these traditional magic numbers, one will also end up with the neutron-deficient nuclei ^{48}Ni, ^{56}Ni, and ^{100}Sn, neutron-rich nuclei ^{78}Ni and ^{132}Sn, as well as the extreme neutron-rich nucleus ^{70}Ca that is predicted as a giant halo nucleus [5]. It has been shown that the shell structures existing in the single-particle spectra can change in the nuclei far away from the stability line [6]. For example, the $N = 28$ shell-closure disappears when the proton number decreases, and thus the nucleus ^{44}S is found to have prolate-spherical shape coexistence [7] and ^{42}Si is well deformed [8]. Therefore, whether the potentially doubly magic nuclei far away from the stability line are indeed doubly magic is a hot and fundamental topic.

Since ^{56}Ni was confirmed as a radioactive doubly magic nucleus on the neutron-deficient side [9], the potentially doubly magic nucleus ^{132}Sn on the neutron-rich side has been paid much attention both experimentally and theoretically in the past years. However, pinning down whether ^{132}Sn is doubly magic or not was not so straight forward. By using the Sn(α,t) reactions, it was found that outside the $Z = 50$ core the energy gap between the proton single-particle states $1h_{11/2}$ and $1g_{7/2}$ increases with neutron excess [10], which suggests a decrease in the nuclear spin-orbit interaction. This was reported in *Nature* in an article entitled *Not-so-magic numbers* [11]. Very recently, by using the ^{132}Sn(d,p)^{133}Sn reaction, the experiment performed at Oak Ridge National Laboratory revealed for the first time that the spectroscopic factors of the neutron single-particle states $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$, and $2f_{7/2}$ outside the $N = 82$ core are consistent with $S \approx 1$. This is one of the critical pieces of evidence to conclude that ^{132}Sn is perfectly doubly magic, even better than ^{208}Pb, which has attracted vast attention [12, 13].

The single-particle states outside a hard double-closure core are crucial for calibrating the theoretical models, especially pure mean field theories, since these valance states are well isolated from the core, and not fragmented. As the only confirmed neutron-rich doubly magic nucleus so far, ^{132}Sn does show unique characteristics in the single-particle spectra. In Fig. 1, the neutron effective single-particle energies (SPE) with respect to the orbital $2f_{7/2}$ are shown. The experimental SPE [14] shown in the left row are compared with those calculated by the Nilsson model with traditional spin-orbit and orbit-orbit parameters [15] shown in the right row. It can be clearly seen that, even though the Nilsson model has achieved great success in describing the single-particle structure for stable nuclei, the traditional parameters encounter a serious problem in reproducing the single-particle spectrum of such neutron-rich nucleus. In other words, the single-particle spectra of exotic nuclei are quite different from those of stable nuclei. Zhang *et al.* pointed out that the Nilsson spin-orbit parameter $C = -2\kappa\hbar\omega$ and the orbit-orbit parameter $D = -\kappa\mu\hbar\omega$ should be re-fitted in this case [16]. Since the experimental SPE has been changed compared to those used in Ref. [16], the parameters C and D are re-fitted accord-

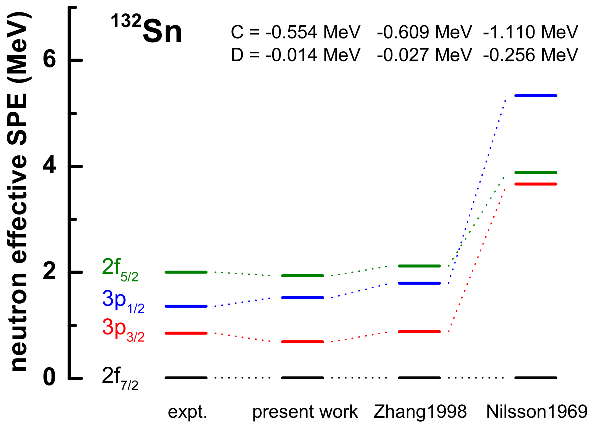

FIG. 1: (Color online) Neutron effective single-particle energies (SPE) with respect to the orbital $2f_{7/2}$ in ^{132}Sn. From left to right, the experimental SPE [14] and those calculated by the Nilsson model with spin-orbit C and orbit-orbit D parameters re-fitted in the present work as well as the parameters given in Ref. [16] and the traditional ones [15] are shown.

ing to the latest experimental SPE in ^{132}Sn in the present work, as shown in Fig. 1. It turns out that the re-fitted spin-orbit parameter C reduces 50%, and the orbit-orbit parameter D is of one order of magnitude smaller than the traditional one.

The relativistic mean field (RMF) theory [4], which has received wide attention due to its successful description of a large variety of nuclear phenomena during the past three decades, is regarded as one of the best microscopic candidates for the descriptions of both stable and exotic nuclei. In this framework, the combination of the scalar and vector fields, which are of the order of a few hundred MeV, provide a natural and more efficient description of both the nuclear mean field central and spin-orbit potentials. So far, the most widely used RMF framework is based on meson-exchange or point-coupling effective interactions. In each case, a quantitative treatment of nuclear matter and finite nuclei needs a medium dependence of effective mean-field interactions, which can be taken into account by the inclusion of higher-order (non-linear coupling) interaction terms or by assuming a density dependence for the coupling interactions.

In this paper, various versions of the RMF theory will be applied to investigate the properties of the neutron-rich doubly magic nucleus ^{132}Sn and the corresponding isotopes and isotones. The two-neutron and two-proton separation energies, the single-particle spectrum, as well as the spin-orbit and orbit-orbit strengths thus extracted will be discussed.

In Fig. 2, the two-neutron separation energies S_{2n} of the Sn isotopes and the two-proton separation energies S_{2p} of the $N = 82$ isotones are shown in the upper and lower panels, respectively. The mass data of ^{132}Sn and ^{134}Sn are taken from Ref. [21] and the others from Ref. [22]. The theoretical results are calculated

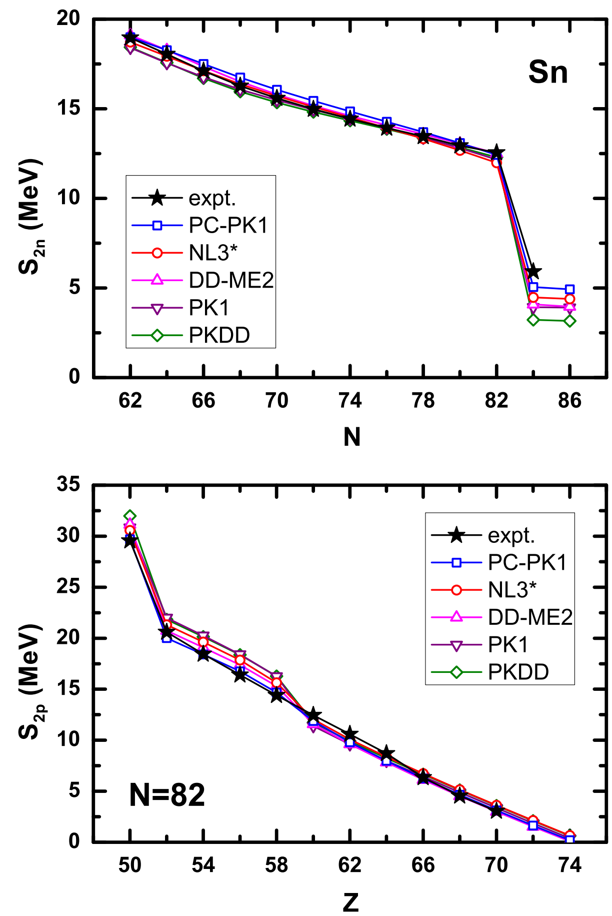

FIG. 2: (Color online) Two-neutron (upper panel) and two-proton (lower panel) separation energies of the Sn isotopes and $N = 82$ isotones calculated by the RMF theory with the effective interactions PC-PK1 [17], NL3* [18], DD-ME2 [19], PK1 [20], and PKDD [20]. The mass data of ^{132}Sn and ^{134}Sn are taken from Ref. [21], and the others from Ref. [22].

by the RMF theory with non-linear point-coupling effective interaction PC-PK1 [17], and non-linear finite-range interactions NL3* [18] and PK1 [20], as well as density-dependent finite-range interactions DD-ME2 [19] and PKDD [20]. It is shown that the experimental data can be well reproduced. In particular, all theoretical results show profound jumps in S_{2n} and S_{2p} at $N = 82$ and $Z = 50$, which indicates that ^{132}Sn is predicted as a legitimate doubly magic nucleus by all the effective interactions used here. It is also found that the size of the neutron shell gaps are slightly overestimated due to the relatively low effective mass. Another critical test of the shell-closure is to study single-particle spectra, especially the single-particle states outside the rigidly spherical core, which can be measured precisely in experiment.

In Fig. 3, the neutron effective SPE with respect to the orbital $2f_{7/2}$ in ^{132}Sn calculated by the RMF theory with effective interactions PC-PK1, NL3*, DD-ME2, PK1, and PKDD are compared with the experimental

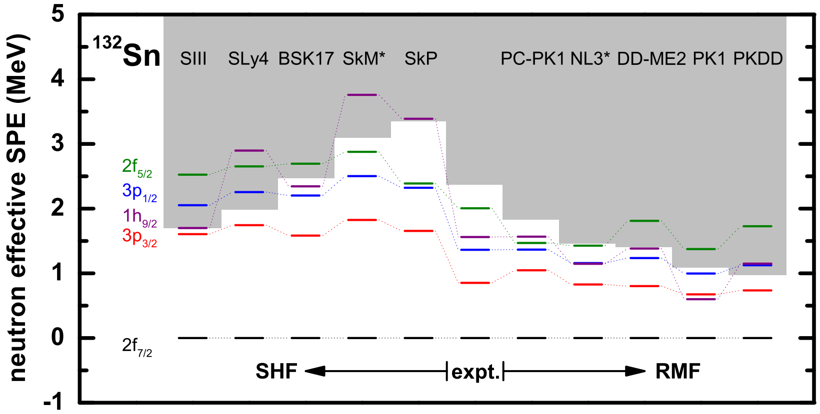

FIG. 3: (Color online) Neutron effective SPE with respect to the orbital $2f_{7/2}$ in ^{132}Sn calculated by the RMF theory with effective interactions PC-PK1, NL3*, DD-ME2, PK1, and PKDD. The experimental data [14] and the results calculated by the Skyrme-Hartree-Fock (SHF) theory with effective interactions SkP [23], SkM* [24], BSK17 [25], SLy4 [26], and SIII [27] are also shown for comparison. The shaded areas indicate the area beyond the neutron threshold.

data [14], where the shaded areas indicate the area beyond the neutron threshold. The results calculated by the Skyrme-Hartree-Fock (SHF) theory with typical effective interactions SkP [23], SkM* [24], BSK17 [25], SLy4 [26] and SIII [27] are also shown for comparison. It should be noted that the overflow of some single-particle states beyond the neutron threshold results from the low effective nucleon mass. An additional enhancement of the effective mass in finite nuclei can be caused by the coupling of single-nucleon levels to low-energy collective vibrational states, which is an effect entirely beyond mean field. In the present mean-field level, it is clearly shown that the overall structure of the neutron effective SPE for such a neutron-rich doubly magic nucleus can be well reproduced in the relativistic framework, where both the spin-orbit and orbit-orbit potentials are described in a self-consistent way. In contrast, the non-relativistic results overestimate nearly twice the single-particle energy spacing between the orbitals $3p_{3/2}$ and $2f_{7/2}$, where the reduction of the orbit-orbit potential plays the most important role.

From the single-particle spectrum, one can extract the Nilsson parameters by fitting to the single-particle energies. The spin-orbit strength C and orbit-orbit strength D are two important characteristics in the analysis of the single-particle spectrum. The Nilsson parameters C and D here are obtained by fitting to the single-particle energies of the states $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$ and $2f_{7/2}$ with the least-squares method.

In order to investigate the evolutions from stable to the neutron-rich nuclei, the Nilsson spin-orbit and orbit-orbit parameters for the $N = 82$ isotonic chain thus obtained are shown in the upper and lower panels of Fig. 4, respectively. The results are shown only for the nuclei in which all four states $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$ and $2f_{7/2}$ are bound. The re-fitted Nilsson parameters for the latest experimental data of ^{132}Sn are plotted with stars. It is noticed that only the data of ^{132}Sn are shown here, since the

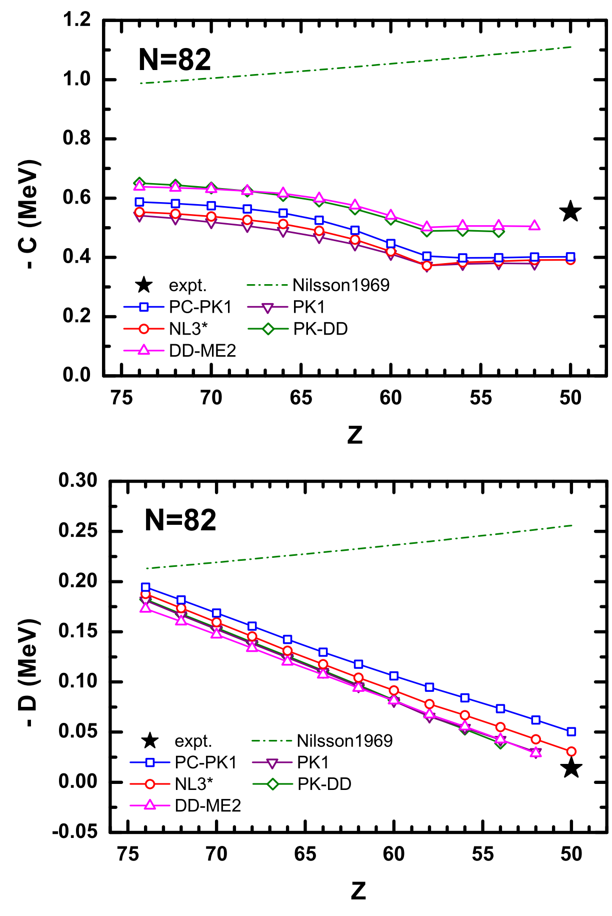

FIG. 4: (Color online) Nilsson spin-orbit (upper panel) and orbit-orbit (lower panel) parameters for the $N = 82$ isotonic chain. Those extracted with the experimental data and RMF calculations are labeled with solid and open symbols, respectively. The traditional parameters from Ref. [15] are shown with dash-dotted lines.

fragmentation in the single-particle states is not negligible for the nuclei that are not doubly magic. In addition, the traditional parameters taken from Ref. [15] are also shown with dash-dotted lines for comparison.

It is shown that the absolute value of the spin-orbit strength C slightly decreases with neutron excess. Its strength for the neutron-rich doubly magic nucleus can be well reproduced by the density-dependent effective interactions, while it is somewhat underestimated by the non-linear interactions. The traditional Nilsson spin-orbit strength shows a slightly escalating trend, which deviates from the data at ^{132}Sn significantly.

Meanwhile, the absolute value of orbit-orbit strength D strength monotonously decreases as the proton number decreases, which shows entirely the opposite tendency of the traditional Nilsson parameters. It is also found that the orbit-orbit strengths extracted from RMF calculations agree quite well with the experimental data for ^{132}Sn, which is greatly overestimated in comparison with the traditional Nilsson parameter.

Therefore, the spin-orbit and orbit-orbit strengths of the neutron-rich doubly magic nucleus ^{132}Sn can be well described by the mean field theories within the relativistic framework. Moreover, the reduction of both the spin-orbit and orbit-orbit strengths along the $N = 82$ isotonic chain are predicted. The reduction shown in this neutron-rich region plays an important role in the calculations using the Nilsson parameters, for example, the yrast bands in ^{136}Te and ^{142}Xe calculated by the projected shell model [16].

In summary, various versions of the RMF theory have been applied to investigate the properties of the neutron-rich doubly magic nucleus ^{132}Sn and the corresponding isotopes and isotones. In particular, the single-particle spectrum as well as the spin-orbit and orbit-orbit strengths thus extracted have been discussed in detail. It is found that the two-neutron and two-proton separation energies are well reproduced by the RMF theory. Moreover, the RMF results agree quite well with the experimental single-particle spectrum in ^{132}Sn as well as the Nilsson spin-orbit parameter C and orbit-orbit parameter D thus extracted, but remarkably differ from the traditional Nilsson parameters. Along the $N = 82$ isotonic chains, the present results show entirely different tendency in comparison with the traditional Nilsson parameters, and provide a guideline for the isospin dependence of the Nilsson parameters.

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